Tutorial on Optoelectronic Oscillators

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Introduction

An optoelectronic oscillator is an RF (radio-frequency) oscillator that incorporates an optical section to enhance loop delay and reduce phase noise. This type of oscillator shows promise in generating low-noise high-frequency microwave signals, which are crucial for applications such as 5G communication, test and measurement equipment, and remote sensing. In this document, we will discuss the structure and operation principles of optoelectronic oscillators. We will also present a mathematical framework, originally derived by X. Steve Yao and Lute Maleki at CalTech [1], that describes the oscillation frequency, amplitude, and noise spectrum of optoelectronic oscillators.

Structure and Working Principles of Optoelectronic Oscillators

The structure of an optoelectronic oscillator is depicted in Figure 1 [1], comprising an optical section and an RF section. The optical section consists of a pump laser, a Mach-Zehnder modulator (MZM), a fiber delay line, and a photodetector. The RF section includes an RF amplifier, an RF coupler, and an RF filter, forming an open-looped RF oscillator. The output of this oscillator is fed back to the modulator, upconverted to an optical frequency, delayed, and then downconverted back to an RF signal by the photodetector, completing the RF loop.

The key distinction between an optoelectronic oscillator and a conventional RF oscillator is the use of an optical section to provide a significant loop delay time (τ) and thus reduce the phase noise of the RF signal ($S_\phi(\omega)$). This large loop delay is made possible by the low-loss nature of the fiber delay line (typically ~0.2 dB/km), a feature unattainable in RF circuits due to the high propagation loss of RF signals in transmission lines (typically ~0.5 dB/cm).

The RF oscillation in the loop is initiated by RF noise originating from the RF amplifier or photodetector. This noise is amplified, filtered to a specific frequency, subjected to an optical delay, and then reintroduced into the RF circuit. The oscillation frequency of the oscillator is dictated by the filter's frequency response, while the oscillation amplitude is influenced by the nonlinearity at the RF amplifier (i.e., gain reduction with increasing input RF power) and the modulator (i.e., generation of harmonics upon large driving voltage). The RF output is extracted at the RF coupler, with the noise spectrum of the output determined by factors such as laser intensity noise, thermal noise, loop delay, photodetector noise, amplifier noise, and filter

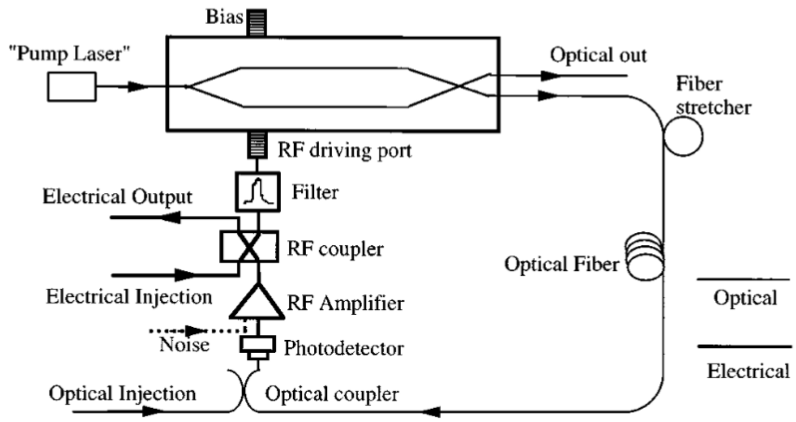

Figure 1: Structure of optoelectronic oscillator.

bandwidth. Subsequent sections will provide a mathematical framework for the quantitative calculation of these parameters.

Oscillation Frequency and Amplitude

To understand the behavior of an optoelectronic oscillator (OEO), we first need to derive its open-loop gain, which is the output voltage at the RF amplifier when a driving signal is applied to the RF driving port. For simplicity, we initially assume the absence of the RF filter and RF coupler, meaning the output of the RF amplifier is not fed back to the RF driving port.

Next, we introduce the RF coupler and filter to connect the RF amplifier to the RF driving port, completing the loop. We then derive the closed-loop gain, defined as the ratio of the output voltage to the input voltage of the RF amplifier. The oscillation frequency of the OEO corresponds to a peak in the closed-loop gain, while the oscillation amplitude is determined by the steady-state condition where the open-loop gain equals unity.

The closed-loop gain of the OEO is the product of the response functions of each component. Given an input RF signal to the RF driving port of the MZM: $V_{\text{in}}(t) = V_o \sin(\omega t + \beta)$, the modulator produces a modulated light accordingly. The intensity of this modulated light oscillates at multiple frequencies, and each Fourier component converts to an RF signal at a specified frequency at the photodetector. The output RF signal after the RF amplifier has a Fourier component at ω with an amplitude of:

$$V_{\text{out}}(t) = G(V_o) V_{\text{in}}(t), \tag{1}$$

where the voltage-gain coefficient $G(V_o) = G_S \frac{2V_\pi}{\pi V_o} J_1(\frac{\pi V_o}{V_\pi})$. Here, V_π is the half-wave voltage, J_1 is the Bessel function, and G_S is the small-signal open-loop gain of the OEO without the RF filter. The Bessel function arises from the Fourier expansion of the modulated light amplitude. Note that G_S depends on the bias voltage of the modulator V_B, the amplifier gain G_A, and the responsivity of the photodetector R.

Equation 1 represents the open-loop small-signal gain without the RF filter. The general expression for open-loop gain also considers the RF filter's frequency-dependent transmission. We use a normalized complex response function to represent the lumped frequency dependence of the open-loop gain:

$$\tilde{F}(\omega) = F(\omega) \exp[i\phi(\omega)]. \tag{2}$$

Incorporating the frequency-dependent transmission of the RF filter into $\tilde{F}(\omega)$, we modify the expression for the open-loop gain to account for the RF filter and loop delay:

$$\tilde{V}_{\text{out}}(t) = \tilde{F}(\omega) G(V_o) \tilde{V}_{\text{in}}(\omega, t), \tag{3}$$

where both $\tilde{V}_{\text{in}}$ and $\tilde{V}_{\text{out}}$ are complex input and output voltages. We assume that all harmonics originating from nonlinear effects are filtered out by the RF filter, ensuring $\tilde{V}_{\text{out}}$ shares the same oscillation frequency as $\tilde{V}_{\text{in}}(\omega, t)$.

Having formulated the expression for the open-loop gain, we now use the recurrence relation of the fields in the loop to derive an expression for the closed-loop gain. An RF signal can circulate the OEO loop multiple times, with the amplitude of the RF signal in the n-th roundtrip related to that in the previous roundtrip through the open-loop gain:

$$\tilde{V}_n(\omega, t) = \tilde{F}(\omega) G(V_o) \tilde{V}_{n-1}(\omega, t) \exp(i\omega\tau'), \tag{4}$$

where τ' denotes the loop delay time of the RF signal due to the physical length of the oscillator. This equation is known as the recurrence relation. The superposition of all recurrence fields builds up the output voltage of the OEO. The input voltage of the modulator $\tilde{V}(\omega, t)$ is related to the input noise (fundamental noise) of the amplifier $\tilde{V}_{\text{in}}(\omega)$ by:

$$\tilde{V}(\omega, t) = \sum_{n=0}^{\infty} \tilde{V}_n(\omega, t) = \frac{G_A \tilde{V}_{\text{in}}(\omega, t)}{1 - \tilde{F}(\omega) G(V_o) \exp(i\omega\tau')} = \frac{G_A \tilde{V}_{\text{in}}(\omega) \exp(i\omega t)}{1 - \tilde{F}(\omega) G(V_o) \exp(i\omega\tau')}, \tag{5}$$

where G_A is the gain coefficient of the amplifier, $\tilde{V}_{\text{in}}(\omega)$ is the complex voltage amplitude of the noise, and τ is the loop delay. This noise amplitude corresponds to a power spectral density of:

$$P(\omega) = \frac{|\tilde{V}(\omega, t)|^2}{2R} = \frac{G_A^2 |\tilde{V}_{\text{in}}(\omega)|^2 / 2R}{1 + |F(\omega) G(V_o)|^2 - 2F(\omega)|G(V_o)| \cos(\omega\tau' + \phi(\omega) + \phi_0)}, \tag{6}$$

where R is the impedance of a test load, and $\phi_0 = 0$ if $G(V_o) > 0$ and $\phi_0 = \pi$ if $G(V_o) < 0$.

While Equation 6 contains the unknown term $G(V_o)$, preventing direct quantitative calculation of the power spectral density, it still provides qualitative insights. Equation 6 suggests that the oscillation amplitudes in the OEO peak at a series of frequencies: $\omega_k \tau' - \phi(\omega_k) + \phi_0 = 2k\pi,\ k \in \mathbb{Z}$, where k is the oscillation mode number. These frequencies correspond to constructive interference of recurrence fields and are potential oscillation frequencies. To achieve single-mode oscillation, a narrow-band RF filter can be used to allow only a single frequency component ω_k to pass. In this case, the oscillation frequency of the OEO is determined by:

$$\omega_{\text{osc}} = \arg\max_{\omega} F(\omega) G_S(\omega) \ \ s.t.\ \omega\tau' + \phi(\omega) + \phi_0 = 2k\pi \ \ (k \in \mathbb{Z}). \tag{7}$$

In other words, the oscillation frequency is the resonance frequency of the OEO loop that maximizes the open-loop gain.

The oscillation amplitude of the OEO can be derived from the steady-state condition where the open-loop gain equals unity: $|G(V_{\text{osc}})| F(\omega_{\text{osc}}) = 1$. Assuming that the RF filter has a 100% transmission at the oscillation frequency, $F(\omega_{\text{osc}}) = 1$, we recall the expression for the open-loop gain and obtain the following equation for oscillation amplitude:

$$|J_1(\frac{\pi V_{\text{osc}}}{V_\pi})| = \frac{\pi V_{\text{osc}}}{2|G_S|V_\pi}. \tag{8}$$

The oscillation amplitude and power can be numerically solved from Equation 8. This equation indicates that the amplitude of the oscillation is influenced by the parameters of the modulator, specifically the half-wave voltage V_π and the small-signal open-loop gain G_S.

Noise Spectrum and Linewidth

In this section, we will derive the noise spectral density of the OEO and present a simplified expression for the linewidth. In deriving Equation 6, we assumed that the input noise of the RF amplifier comprises discrete Fourier components. However, in practice, the input noise has a continuous spectrum, denoted as $\rho_N(\omega)$. The noise power within a bandwidth Δf is $\rho_N(\omega)\Delta f$. This noise within the bandwidth can be treated as a discrete Fourier component with amplitude $\bar{V}_{\text{in}}(\omega)$, where $|\bar{V}_{\text{in}}(\omega)|^2/2R = \rho_N(\omega)\Delta f$. Using Equation 6, we can rewrite the noise power spectral density as:

$$S_{\text{RF}}(f') = \frac{P(f')}{\Delta f P_{\text{osc}}} = \frac{G_A^2 \rho_N(f)/P_{\text{osc}}}{1 + |F(f')G(V_{\text{osc}})|^2 - 2F(f')|G(V_{\text{osc}})|\cos(2\pi f'\tau)}, \tag{9}$$

where f' is the frequency offset from the oscillation frequency f_{osc}, and $\tau = \tau' + \frac{d\phi(\omega)}{d\omega}\big|_{\omega=\omega_{\text{osc}}}$ is the total group delay, including the real-time delay from the loop's physical length and the group delay from dispersive components. In deriving Equation 9, we assumed that the open-loop gain at frequency $f_{\text{osc}} + f'$ is equal to that at the oscillation frequency f_{osc}. This assumption holds when f' is much smaller than the free spectral range (FSR) of the OEO, $1/\tau$.

The noise spectral density $S_{\text{RF}}(f')$ can be greatly simplified under certain assumptions and approximations (see detailed derivation in [1], Equations 21-24). It can be shown that, when $2\pi f'\tau \ll 1$, the noise spectral density can be approximately expressed as:

$$S_{\text{RF}}(f') = \frac{\delta}{(\delta/2\tau)^2 + (2\pi)^2(\tau f')^2}, \tag{10}$$

where $\delta = \rho_N G_A^2/P_{\text{osc}}$ represents the input noise-to-signal ratio measured at the amplifier's output. Equation 10 suggests that if the frequency dependence of the input noise δ can be neglected, the OEO's output will have a Lorentzian spectral shape. The corresponding full width at half maximum (FWHM) linewidth is:

$$\Delta f_{\text{FWHM}} = \frac{\delta}{2\pi\tau^2} = \frac{1}{2\pi}\frac{G_A^2\rho_N}{\tau^2 P_{\text{osc}}}. \tag{11}$$

Therefore, the linewidth of an OEO is inversely proportional to the square of the loop delay time τ and linearly dependent on the input noise-to-signal ratio δ.

Approaches to Low-noise Optoelectronic Oscillators

To achieve a low-noise Optoelectronic Oscillator (OEO) with minimal linewidth, one can employ a long loop delay, denoted as τ. There are two main approaches to realize this:

Approach 1: Long Fiber Delay Line

The first approach involves inserting a long fiber delay line after the modulator. This increases the physical length of the loop, thus extending the true time delay (τ') of the signal. This method has been widely used in previous demonstrations of OEOs, enabling the generation of ultralow-noise microwave signals (e.g., phase noise = -158 dBc/Hz at a 10 kHz frequency offset and a 10 GHz carrier) [2]. Typically, these demonstrations require a loop delay time of 1 microsecond or longer, necessitating a bulky fiber-optic setup. In contrast, typical integrated true time delays are on the order of several nanoseconds, limited by lithography area, which may not suffice for building low-noise OEOs.

Approach 2: Group Delay of Dispersive Components

The second approach leverages the group delay of dispersive components, which is promising for compact realization and potential integration. Specifically, given modulated light with a carrier frequency ω_c and modulation frequency ω_{osc}, a microring resonator with resonance peaks at $\omega_c + k\omega_{\text{osc}}$ ($k \in \mathbb{Z}$) can achieve a long group delay of the modulated light. The key requirement for this approach is that the microring resonator's Free Spectral Range (FSR) must be divisible by the OEO oscillation frequency. As long as all frequency components of the modulated light experience the same group delay, the RF envelope of the light will gain a time delay without altering its waveform.

Below, we present the calculation of the group delay of the RF waveform induced by a microring resonator in an all-pass filter configuration. We assume the resonator has a resonance at ω_c and an FSR of ω_{osc}. Provided a modulated light input:

$$E_{\text{input}} = (1 + \eta\cos(\omega_{\text{osc}}t))\exp(i\omega_c t),$$

where η is a unitless coefficient representing modulation depth. The input modulated light has three frequency components located at ω_c, $\omega_c + \omega_{\text{osc}}$, and $\omega_c - \omega_{\text{osc}}$, respectively:

$$E_{\text{input}} = \exp(i\omega t) + \frac{\eta}{2}\exp(i(\omega_c + \omega_{\text{osc}})t) + \frac{\eta}{2}\exp(i(\omega_c - \omega_{\text{osc}})t). \tag{12}$$

The amplitude transmission of the microring all-pass filter is given by [4]:

$$t(\omega) = e^{i(\pi+\beta L)}\frac{a - re^{-i\beta L}}{1 - rae^{i\beta L}}, \tag{13}$$

where $\beta = \beta(\omega)$ is the propagation constant, L is the circumference of the microring, a is the amplitude roundtrip transmission, and r is the amplitude self-coupling coefficient. Since all three frequency components in E_{input} satisfy the resonance condition of the microring ($\beta L = 2k\pi$ for some $k \in \mathbb{Z}$), we have $t(\omega_c) = t(\omega_c + \omega_{\text{osc}}) = t(\omega_c - \omega_{\text{osc}}) = (r - a)/(1 - ra)$. The amplitude of the transmitted light thus follows:

$$E_{\text{pass}} = t(\omega_c)e^{i\omega_c t} + \frac{\eta}{2}t(\omega_c + \omega_{\text{osc}})e^{i(\omega_c+\omega_{\text{osc}})t} + \frac{\eta}{2}t(\omega_c - \omega_{\text{osc}})e^{i(\omega_c-\omega_{\text{osc}})t} = \frac{r-a}{1-ra}E_{\text{input}}.$$

Therefore, the microring resonator preserves the waveform of the modulated light. This feature relies on the OEO oscillation frequency being a multiple of the microring's FSR. Next, we evaluate the group delay of the RF signal modulated onto the optical carrier. To this end, we need to calculate the phase delay of the envelope function when there is a perturbation to the OEO oscillation frequency $\delta\omega$. In this case, the input modulated light has an amplitude of

$$E_{\text{input}} = e^{i\omega_c t}[1 + \eta\cos((\omega_{\text{osc}} + \delta\omega)t)].$$

The corresponding transmitted light amplitude is:

$$E_{\text{pass}} = t(\omega_c)e^{i\omega_c t} + \frac{\eta}{2}(t(\omega_c) + t'(\omega_c)\delta\omega)e^{i(\omega_c+\omega_{\text{osc}}+\delta\omega)t} + \frac{\eta}{2}(t(\omega_c) - t'(\omega_c)\delta\omega)e^{i(\omega_c-\omega_{\text{osc}}-\delta\omega)t}.$$

We have used the periodicity of $t(\omega)$ when deriving this equation. Now we recast this equation to make the envelope explicit:

$$E_{\text{pass}} = t(\omega_c)e^{i\omega_c t}[1 + \eta(\cos(\omega_{\text{osc}} + \delta\omega)t + i\frac{t'(\omega_c)}{t(\omega_c)}\delta\omega\sin(\omega_{\text{osc}} + \delta\omega)t)]. \tag{14}$$

Note that ω_c is a resonance frequency of the microring, where $|t(\omega)|$ reaches its minimum value and $|t|'(\omega_c) = 0$. Let's denote $t(\omega) = |t|(\omega)\exp(i\phi(\omega))$, then:

$$t'(\omega_c) = |t|'(\omega_c)e^{i\phi(\omega_c)} + i|t(\omega_c)|e^{i\phi(\omega_c)}\phi'(\omega_c) = it(\omega_c)\tau_c, \tag{15}$$

where $\tau_c = \phi'(\omega_c)$ is the group delay of the carrier wave induced by the microring. Substitute Equation 15 into Equation 14, we have

$$\begin{aligned} E_{\text{pass}} &= t(\omega_c)e^{i\omega_c t}[1 + \eta(\cos(\omega_{\text{osc}} + \delta\omega)t - \delta\omega\tau_c\sin(\omega_{\text{osc}} + \delta\omega)t)] \\ &\approx t(\omega_c)e^{i\omega_c t}[1 + \eta\cos((\omega_{\text{osc}} + \delta\omega)t + \delta\omega\tau_c)]. \end{aligned} \tag{16}$$

This equation shows that, under a first-order approximation, the microring resonator can preserve the waveform of the modulated light even if the modulation frequency slightly deviates from a multiple of the microring's FSR. Additionally, the microring induces the same group delay in the modulation waveform as the group delay of the carrier wave. Consequently, we can use a photonic microring resonator to create a group delay for RF signals modulated onto an optical carrier wave, provided the microring's FSR is a multiple of the RF signal's frequency and the carrier frequency aligns with a microring resonance. This approach facilitates the design of compact RF delay lines and the development of integrated optoelectronic oscillators.

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