Derivation of ring enhancement factor

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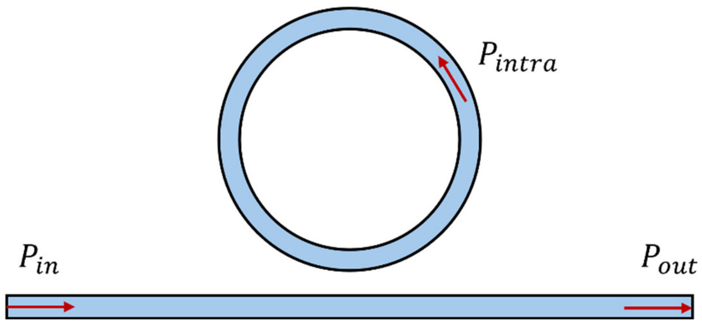
This Note serves to derive the ring enhancement factor, or the ratio of intra-cavity power to the input power on the bus waveguide, for an all-pass ring resonator.



Problem description:
Given input power P_{in}, intrinsic and loaded Q factors of the resonator Q_i and Q, calculate the intra-cavity power P_{intra}.

Solution:
The power transmission of the ring at a resonant frequency is given by

$$T_{res} = \frac{(a-r)^2}{(1-ra)^2},$$

where a is roundtrip amplitude transmission coefficient, and r is self-coupling coefficient.

The loaded, intrinsic, and coupling Q factors of the all-pass ring are given by:

$$Q = \frac{\pi n_g L\sqrt{ra}}{\lambda_{res}(1-ra)} \approx \frac{\pi n_g L}{\lambda_{res}(1-ra)},$$

$$Q_i = \frac{\pi n_g L\sqrt{a}}{\lambda_{res}(1-a)} \approx \frac{\pi n_g L}{\lambda_{res}(1-a)},$$

$$Q_c = \frac{\pi n_g L\sqrt{r}}{\lambda_{res}(1-r)} \approx \frac{\pi n_g L}{\lambda_{res}(1-r)},$$

where we assume the roundtrip attenuation and coupling are weak ($r, a \approx 1$).

Now the power transmission can be recast into

$$T_{res} = \frac{[(1-r)-(1-a)]^2}{(1-ra)^2} = Q^2\left(\frac{1}{Q_c} - \frac{1}{Q_i}\right)^2$$

Next, we express P_{intra} as a function of T_{res}. By the conservation of energy, we have:

$$P_{in}(1-T_{res}) = P_{diss},$$

Note that

$$\frac{P_{diss}}{f_{res}} = 2\pi \times \frac{\text{intracavity energy}}{Q} = 2\pi \times \frac{P_{intra}}{Q} \times \frac{n_g L}{c}$$

Combine these equations, we have

$$P_{diss} = 2\pi \frac{f_{res} P_{intra} n_g L}{Qc} = P_{in}(1 - T_{res}) = P_{in}\left[1 - Q^2\left(\frac{1}{Q_c} - \frac{1}{Q_i}\right)^2\right]$$

Which can be simplified into

$$\frac{P_{intra}}{P_{in}} = \frac{Qc}{2\pi n_g L f_{res}}\left[1 - \left(\frac{Q}{Q_c} - \frac{Q}{Q_i}\right)^2\right]$$

Example usage:

Let Q_c, Q_i = 50 M, Q = 25 M, $L = 6$ mm, $n_g = 1.6$, f_{res} = 193 THz, P_{in} = 1 mW, we have

$$P_{intra} = 644\ mW$$

Reference:

[1] https://en.wikipedia.org/wiki/Q_factor

[2] Bogaerts, Wim, et al. "Silicon microring resonators." *Laser & photonics reviews* 6.1 (2012): 47-73.